Important Concepts	Examples

Exponential Growth

An exponential pattern of change involves patterns that are based on multiplication and can often be recognized in a verbal description of a situation or in the pattern of change in a table of (*x*, *y*) values.

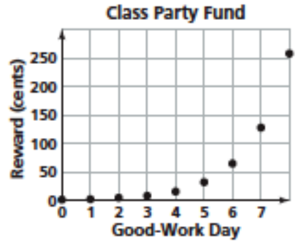
The increasing rate of growth is reflected in the upward curve of the plotted points.

Suppose a reward is offered. At the start, 1¢ is put in a fund. On the first day, 2¢ is added; on the second day, 4¢ is added; and on each succeeding day, the reward is doubled. How much money is added on the eighth day?

Class Party Fund

Good-Work Day	Reward (cents)
0 (start)	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	?

Class Party Fund



Growth Factor

A constant factor can be obtained by dividing each successive *y*-value by the previous *y*-value. This ratio is called the *growth factor* of the pattern.

In the example above, you multiply the previous award by 2 to get the new reward. This constant factor can also be obtained by dividing successive *y*-values: $\frac{2}{1} = 2$, $\frac{4}{2} = 2$, etc.

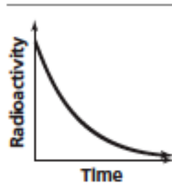
Exponential Equations

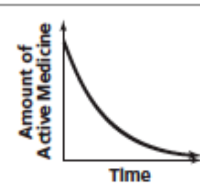
EXPONENTIAL GROWTH An exponential growth pattern, $y = a(b)^x$, increases slowly at first but grows at an increasing rate because its growth is multiplicative. The growth factor is *b*.

Day	Calculation	Reward (cents)
0	1	1
1	1×2, or 2^1	2
2	$1 \times 2 \times 2$, or 2^2	4
3	$1 \times 2 \times 2 \times 2$, or 2^3	8
⋮	⋮	⋮
6	$1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$, or 2^6	256
⋮	⋮	⋮
n	$1 \times 2 \times 2 \times \ldots \times 2$, or 2^n	2^n

On the *n*th day, the reward, R, will be $R = 1 \times 2^n$. Because the independent variable in this pattern appears as an exponent, the growth pattern is called exponential. The growth factor is the *base*, 2. The *exponent*, *n*, tells the number of times the 2 is a factor.

EXPONENTIAL DECAY Exponential models describe patterns in which the value decreases. Decay factors result in decreasing relationships because they are less than 1.





$y = 50(\frac{1}{2})^n$

Rules of Exponents

The multiplicative structure of bases leads to:

$(b^m)^n = b^{mn}$	$(2^3)^2 = (2 \times 2 \times 2)^2 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2^6$
$(b^m)(b^n) = b^{m+n}$	$3^2 \times 3^3 = (3 \times 3) \times (3 \times 3 \times 3) = 3^5 = 243$
$(a^m b^m) = (ab)^m$	$(2 \times 5)^2 = (2 \times 5) \times (2 \times 5) = (2 \times 2) \times (5 \times 5) = 2^2 \times 5^2$
$a^m/a^n = a^{m-n}$	$5^3/5^2 = (5 \times 5 \times 5)/(5 \times 5) = 5^{3-2} = 5^1 = 5$