

Grades 6-8

Mathematics CPS Curriculum Overview

The grade 6-8 mathematics curriculum will solidify students' understanding of fractions, decimals, and percents. A major emphasis of grades seven and eight will be a strong contextual Algebra 1 experience where students will have extensive experiences representing algebraic problems in tables, graphs, and symbolic forms. In addition, students will extend their understandings in statistics and probability and be able to develop and evaluate arguments based on statistical analysis. They will develop solid understandings in context of the use of central tendencies such as mean, median, and mode. They will also develop an appreciation for the pervasive use of probability in the real world, and be able to compare experimental and theoretical probabilities. Students' geometric understanding will be extended to include the ability to describe the properties of three-dimensional figures such as the sphere, prism, pyramid, cylinder, and cone. They will also develop solid understanding of the concepts of surface area and volume. Students will continue their work with patterns by describing, extending, analyzing, and creating arithmetic, geometric, and exponential patterns.

During grades 6-8 student's problems solving skills will be extended to include work with multi-step and non-routine problems with an emphasis on explaining how symbolic, visual, and graphical representations can communicate the same solutions. Students will develop the ability to generalize solutions and strategies to new problem situations. The grade 6-8 mathematics classroom will be rich in mathematical discourse emphasizing all students' oral and written communication skills in justifying their thinking and reasoning.

By the end of grade 7 and grade 8, all students should be able to pass the Algebra 1 final. They should also be competent in the statistics/probability and geometry/measurement learning standards necessary to pass the grade 8 MCAS in mathematics. Most importantly, by the end of grade 8, a student should feel competent as a mathematics learner and should obtain enjoyment and pride from engagement in mathematical endeavors.

Recommended Materials: The Connected Mathematics Program (CMP) is the CPS adopted program for grades 6-8. Teachers are encouraged to use supplementary materials and make adaptations as necessary.

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Mathematics Learning Expectations for Grade 6

I. PROBLEM SOLVING

A. Students will develop the ability to use problem-solving approaches to investigate and understand mathematical content.

They will be able to:

- Apply a variety of strategies to solve a single problem
- Decide which strategy is most efficient for solving a problem
- Read symbolic notation which represents a variety of problem-solving situations
- Begin to write simple expressions to represent problem situations with one variable
- Begin to use formulas in problem-solving situations

B. Students will develop the ability to formulate problems from situations within and outside mathematics.

They will be able to:

- Create mathematical problems that arise from many areas of the curriculum
- Understand the value of mathematical knowledge as a tool for solving real-life problems

C. Students will develop and apply a variety of strategies to solve problems, with emphasis on multi-step and non-routine problems.

They will be able to:

- Decide whether to use mental arithmetic, paper and pencil, or technology to solve a problem
- Use mathematical knowledge from all topics to solve real-life problems

D. Students will develop the ability to verify and interpret results with respect to the original problem situation.

They will be able to:

- Explain the mathematical thinking they used in solving a problem
- Use visual representations to describe their mathematical thinking

E. Students will develop the ability to generalize solutions and strategies to new problem situations.

They will be able to:

- Compile a list of questions to ask in problem-solving situations
- Justify why a certain strategy is more efficient than others in a given situation

F. Students will develop the ability to acquire confidence in using mathematics meaningfully.

They will be able to:

- Begin to take initiative and rely less on the teacher in determining and applying problem-solving strategies

II. NUMBER RELATIONSHIPS

A. Students will develop the ability to understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential and scientific notation) in real-world and mathematical problem situations.

They will be able to:

- Use arithmetic expressions to show the composition of a number
- Use exponential notation to write equivalent arithmetic expressions
- Use simple fraction-decimal-percent equivalences (halves, fourths, fifths, and tenths) in situations such as tipping, sports averages, etc.
- Use the reciprocal of a divisor when dividing by a fraction
- Solve a division example involving decimals by using equivalent forms with a whole divisor (e.g., $8.4 \div .07 = 840 \div 7$)
- Understand and use number systems in a variety of bases

B. Students will develop number sense for whole numbers, fractions, decimals, integers, and rational numbers.

They will be able to:

- Read numbers greater than 999,999
- Explain the system of place value in decimal notation, i.e., base ten
- Compare fractions by finding decimal equivalents
- Distinguish between the way decimals are commonly read and what they mean

C. Students will develop the ability to understand and apply ratios, proportions and percents in a wide variety of situations.

They will be able to:

- Calculate percents as they occur in problem-solving situations
- Identify, read, write and solve ratios and proportions

D. Students will develop the ability to investigate relationships among fractions, decimals and percents

They will be able to:

- Write a decimal, fraction, or percent in the other equivalent forms

E. Students will develop the ability to represent numerical relationships in one- and two-dimensional graphs.

They will be able to:

- Collect, organize and present data on a variety of graphs on paper and in electronic format
- Plot points in four quadrants on a Cartesian Coordinate System

III. SYSTEMS AND NUMBER THEORY

A. Students will develop the ability to understand and appreciate the need for numbers beyond the whole numbers.

They will be able to:

- Use simple fractions, both proper and improper, to represent parts of a whole

B. Students will develop and use order relations for whole numbers, fractions, decimals, integers, and rational numbers.

They will be able to:

- Order whole numbers greater than 999,999
- Order fractions as small as thousandths

- Order positive and negative numbers

C. Students will develop the ability to extend their understanding of whole number operations to fractions, decimals, integers, and rational numbers

They will be able to:

- Perform the four operations with fractions. Add, subtract, multiply, and divide decimal fractions

D. Students will develop the ability to understand how the basic arithmetic operations are related to one another.

They will be able to:

- Solve problems in which basic operations can be interchanged
- Validate solutions to problems using a related operation

E. Students will develop and apply number theory concepts, e.g., primes, factors, and multiples) in real-world and mathematical problem situations.

They will be able to:

- Develop and expand their understanding of place value through examination of other number systems and bases
- Find the least common multiple and greatest common factor of 2 or more numbers

IV. COMPUTATION AND ESTIMATION

A. Students will develop the ability to compute with whole numbers, fractions, decimals, integers, and rational numbers.

They will be able to:

- Use an efficient algorithm for multiplication (up to 3-digit numbers) and division of whole numbers with 2-digit divisors
- Add and subtract simple fractions and mixed numbers with unlike denominators

- Convert mixed numbers to improper fractions and vice versa

- Simplify fractions

- Multiply mixed numbers

- Divide simple fractions and mixed numbers

- Multiply decimals

- Divide whole numbers and decimals by whole number and decimal divisors

- Recognize perfect squares

- Find the percent of any number

B. Students will develop, analyze, and explain procedures for computation and techniques for estimation.

They will be able to:

- Apply divisibility rules for 2, 3, 4, 5, 6 and 9 to simplify fractions

- Demonstrate relationships among fractions, decimals, and percents by converting among them

- Apply the rules for the order of operations

- Estimate products and quotients for whole numbers and decimals

C. Students will develop, analyze, and explain methods for solving proportions.

They will be able to:

- Write a proportion based on a problem-solving situation

D. Students will select and use an appropriate method for computing from among mental arithmetic, paper-and-pencil, calculator, and computer methods.

They will be able to:

- Decide if an exact answer is required, or if an estimate is sufficient, given a problem situation

- Once this determination is made, decide which tool is most efficient -- mental arithmetic, paper and pencil, calculator, or computer.

E. Students will use computation, estimation, and proportions to solve problems.

They will be able to:

- Enlarge and reduce the sizes of objects according to scale factors

F. Students will use estimation to check the reasonableness of results.

They will be able to:

- Validate answers by checking actual solutions against estimates

V. PATTERNS AND FUNCTIONS

A. Students will describe, extend, analyze, and create a wide variety of patterns.

They will be able to:

- Identify and explain how word and symbol patterns reflect the underlying structure of the number system
- Invent patterns
- Identify patterns originating from numbers
- Relate visual and numeric patterns
- Find and describe patterns, and test whether a pattern always holds true
- Recognize and compare linear and exponential patterns
- Recognize patterns in an arithmetic, geometric or exponential sequence

B. Students will describe and represent relationships with tables, graphs, and rules.

They will be able to:

- Describe patterns with concrete objects, and then represent them with tables, graphs, or rules on paper and in an electronic format

C. Students will analyze functional relationships to explain how a change in one quantity results in a change in another.

They will be able to:

- Explore a problem in which two variables determine a pattern

D. Students will use patterns and functions to represent and solve problems.

They will be able to:

- Test whether patterns are true in different situations
- Determine when to use each of four types of representations (words, tables, graphs, equations) in solving a given problem

VI. ALGEBRA

A. Students will understand the concepts of variable, expression, and equation.

They will be able to:

- Write a linear equation to represent a pattern

B. Students will represent situations and number patterns with tables, graphs, verbal rules, and equations and explore the interrelationships of these representations.

They will be able to:

- Construct an input/output table for a given rule of an arithmetic sequence
- Plot points and graph linear equations
- Use tools such as pictures, tables, and graphs for finding, describing and extending patterns
- Recognize pattern-seeking as a skill which can be applied in other situations such as predicting outcomes in science

C. Students will analyze tables and graphs to identify properties and relationships.

They will be able to:

- Describe the data contained on a graph or in a table
- Make predictions based on data from a graph or a table

D. Students will develop confidence in solving linear equations using concrete, informal and formal methods.

They will be able to:

- Solve linear equations of the form $y = x + b$, i.e., find the n th term once the general pattern has been determined
- Use inverse operations in solving equations

E. Students will investigate inequalities and non-linear equations informally.

They will be able to:

- Plot linear and non-linear (quadratic, exponential) equations to determine which change at a steady (arithmetic) rate, which change geometrically, and which change exponentially, e.g., *The Will*

F. Students will apply algebraic methods to solve a variety of real-world and mathematical problems.

They will be able to:

- Explore and represent algebraically relationships that exist in the real world, e.g., Crossing the River
- Recognize that algebra is an extension of arithmetic and geometry
- Solve complex-looking equations by the backtracking method

VII. STATISTICS

A. Students will systematically collect, organize, and describe data on paper and electronically.

They will be able to:

- Formulate key questions to continually narrow the focus of investigations and refine the representation of data
- Develop survey instruments to obtain appropriate data
- Determine how many samples are needed to accumulate enough appropriate data

B. Students will construct, read, and interpret tables, charts, and graphs.

They will be able to:

- Construct a line graph
- Make vertical and horizontal bar graphs
- Record data in a frequency table
- Read *and construct* a circle graph (Note the change!)
- Choose the most efficient way to represent data in bar, circle, or line graphs or in pictographs

C. Students will make inferences and convincing arguments that are based on data analysis.

They will be able to:

- Find the mean, median and mode

D. Students will evaluate arguments that are based on data analysis.

They will be able to:

- Assess how data is collected and set up by other students by examining their hypotheses, supporting information, and conclusions

E. Students will develop an appreciation for statistical methods as powerful means for decision-making.

They will be able to:

- Decide whether or not a mean adequately describes a population or if other statistical measures or descriptions are necessary.

VIII. PROBABILITY

A. Students will model situations by devising and carrying out experiments or simulations to determine probabilities.

They will be able to:

- Refine their understanding of the concept of chance
- Use simulations to model the probability of real-world events
- Apply probability to determine fairness
- Use list and grid methods for determining all possible combinations of events

B. Students will model situations by constructing a sample space to determine probabilities.

They will be able to:

- Construct both a frequency and a strip graph
- Represent probability using decimals, fractions and percents

C. Students will appreciate the power of using a probability model by comparing experimental results with mathematical expectations.

They will be able to:

- Determine theoretical probabilities using outcome grids

D. Students will make predictions that are based on experimental or theoretical probabilities.

They will be able to:

- Apply proportional reasoning to predict outcomes with a large number of tries

E. Students will develop an appreciation for the pervasive use of probability in the real world.

They will be able to:

- Analyze data to determine the probabilities of real world events

- Model the likelihood of actual events occurring in real world situations by creating a simulation

IX. GEOMETRY

A. Students will identify, describe, compare and classify geometric figures.

They will be able to:

- Describe a quadrilateral, including square, rhombus, rectangle, and trapezoid in terms of its attributes
- Classify polygons by sides and angles
- Identify equal and parallel sides and equal and right angles

B. Students will visualize and represent geometric figures with special attention to developing spatial sense.

They will be able to:

- Identify the attributes of a quadrilateral from a drawing
- Make isometric and orthogonal drawings of 3-D shapes
- Represent three dimensional structures in two dimensions from a variety of viewpoints

C. Students will explore transformations of geometric figures.

They will be able to:

- Visualize and construct 3-D geometric figures using a variety of polygons

D. Students will represent and solve problems using geometric models.

They will be able to:

- Use area models to solve arithmetic problems involving decimals, proportions, and percents

E. Students will develop the ability to understand and apply geometric properties and relationships.

They will be able to:

- Given a set perimeter of a quadrilateral, maximize the area
- Calculate the sum of the angles of a triangle or a polygon

F. Students will develop an appreciation of geometry as a means of describing the physical world.

They will be able to:

- Recognize that structures are collections of various polyhedra

X. MEASUREMENT

A. Students will extend their understanding of the process of measurement.

They will be able to:

- Use the appropriate system of measurement in a real-life problem situation, i.e., length, area, volume, capacity, mass
- Understand the concept of scaling and be able to rescale
- Use fractions when scaling or rescaling

B. Students will estimate, make, and use measurements to describe and compare phenomena..

They will be able to:

- Estimate and measure the dimensions of objects
- Solve problems in one, two and three dimensions

C. Students will select appropriate units and tools to measure to the degree of accuracy required in a particular situation.

They will be able to:

- Use square and cubic units to measure area and volume
- Use a protractor to measure angles
- Create and interpret scale drawings

D. Students will understand the structure and use of systems of measurement.

They will be able to:

- Measure with inches, feet, and fractions of inches
- Compare the U.S. customary and metric systems of measurement
- Find the scale factor that describes the relationship between sizes

E. Students will extend their understanding of the concepts of perimeter, area, volume, angle measure, capacity, and weight and mass.

They will be able to:

- Solve problems involving area and volume
- Estimate area and volume
- Explore the effects of rescaling on area and volume

F. Students will develop the concepts of rates and other derived and indirect measurements.

They will be able to:

- Use measurement conversions (centimeters to meters) in the problem-solving process

G. Students will develop formulas and procedures for determining measures to solve problems.

They will be able to:

- Approximate the areas of irregular polygons, using smaller squares and/or rectangles

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Mathematics Learning Expectations for Grade 7

I. PROBLEM SOLVING

Students will select from among a variety of strategies the appropriate means to solve a given problem.

A. Students will develop the ability to use problem-solving approaches to investigate and understand mathematical content.

They will be able to:

- Determine the most appropriate strategy for a given problem
- Read symbolic notation that describes a variety of problem-solving situations
- Write simple expressions to represent problem situations with one variable

B. Students will develop the ability to formulate problems from situations within and outside mathematics.

They will be able to:

- Explore and use formulas for various problem-solving situations

C. Students will develop and apply a variety of strategies to solve problems, with emphasis on multi-step and non-routine problems.

They will be able to:

- Use given models or algorithms to solve problems, and explain why they apply
- Decide whether to use mental arithmetic, paper and pencil, or technology

D. Students will develop the ability to verify and interpret results with respect to the original problem situation.

They will be able to:

- Make visual representations that describe problem situations, (i.e., tables, diagrams, graphs, pictures on paper or in electronic format)

E. Students will develop the ability to generalize solutions and strategies to new problem situations.

They will be able to:

- Develop models to describe cross-disciplinary situations (i.e., endangered zebras --future numbers)
- Generate questions related to the problem

F. Students will develop the ability to acquire confidence in using mathematics meaningfully.

They will be able to:

- Demonstrate initiative in solving mathematical problems without relying on constant teacher direction

II. NUMBER RELATIONSHIPS

Students will expand their knowledge of the number system to include scientific notation and rational numbers to solve more sophisticated problems.

A. Students will develop the ability to understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential and scientific notation) in real-world and mathematical problem situations.

They will be able to:

- Use scientific notation to represent large distances, large sums of money, and microscopic phenomena
- Given a problem situation, determine and use the most appropriate form of notation (i.e., fraction, decimal, or percent)

B. Students will develop number sense for whole numbers, fractions, decimals, integers, and rational numbers.

They will be able to:

- Decide when an estimate is sufficient
- Decide when an answer is reasonable (i.e., within a range of possibilities)
- Develop rules for addition, subtraction, multiplication, and division of integers
- Identify effects of multiplication and division on rational numbers

C. Students will develop the ability to understand and apply ratios, proportions and percents in a wide variety of situations.

They will be able to:

- Compare size in terms of percent
- Set up and solve proportions

D. Students will develop the ability to investigate relationships among fractions, decimals and percents

They will be able to:

- Write a decimal, fraction, or percent in the other equivalent forms

E. Students will develop the ability to represent numerical relationships in one- and two-dimensional graphs.

They will be able to:

- Locate or plot a decimal on a number line
- Graph a given inequality on a number line
- Identify exponential growth

- Plot graphs representing arithmetic and geometric sequences
- Execute the above tasks on paper or electronically

III. SYSTEMS AND NUMBER THEORY

Students will extend their understanding of rational numbers as they apply concepts of primes, factors, and multiples to simplify arithmetic and algebraic expressions.

A. Students will develop the ability to understand and appreciate the need for numbers beyond the whole numbers.

They will be able to:

- Formulate an argument to show that squaring a proper fraction does not result in a number that is larger than the original fraction.
- Find examples in which outcomes of problem solving do not result in whole or rational numbers.

B. Students will develop and use order relations for rational numbers (i.e., fractions, decimals, integers)

They will be able to:

- Use scientific notation to determine examples of numbers between two numbers expressed exponentially

C. Students will develop the ability to extend their understanding of whole number operations to fractions, decimals, integers, and rational numbers

They will be able to:

- Perform the four operations with fractions. Add, subtract, multiply, and divide decimal fractions

D. Students will develop the ability to understand how the basic arithmetic operations are related to one another.

They will be able to:

- Recognize in algebraic equations that addition and subtraction are related, as are multiplication and division

- Solve for x given a single step equation (e.g. $3x = 21$), or a proportion (e.g. $3/5 = x/25$)
- Given an equation in the form $x + a = b$, solve by using the equivalent form $x = b - a$; or given $ax = b$, use the form $x = b/a$

E. Students will develop and apply number theory concepts, e.g., primes, factors, and multiples) in real-world and mathematical problem situations.

They will be able to:

- List numbers which have exactly three factors, and determine a relevant pattern (i.e., squares of all primes)
- Recognize powers of ten and predict outcomes in a variety of contexts

IV. COMPUTATION AND ESTIMATION

Students will determine if an estimate or exact answer is required, and then select from among paper and pencil, mental arithmetic, or technology to solve the given problem.

A. Students will develop the ability to compute with rational numbers (i.e. fractions, decimals, integers).

They will be able to:

- Use fraction computation with variables { i.e., $(1/3)x = 15$, $(1/2)x + 6 = 18$ }
- Divide mixed numbers
- Use reciprocals to solve equations with fractional coefficients { i.e., $(1/2)x = 8$ }
- Use all four operations with integers to solve problems derived from real-life situations
- Apply the laws of exponents
- Recognize perfect squares up to 152, and thereby find the square root of any of these numbers

B. Students will develop, analyze, and explain procedures for computation and techniques for estimation.

They will be able to:

- Practice rules for order of operations with integers
- Use scientific notation in real-life problems involving very large numbers (i.e., distances in solar system)

C. Students will develop, analyze, and explain methods for solving proportions.

They will be able to:

- Translate words to algebraic notation when given a proportional problem situation

D. Students will select and use an appropriate method for computing from among mental arithmetic, paper-and-pencil, calculator, and computer methods.

They will be able to:

- Decide if an exact answer is required, or if an estimate is sufficient when given a problem situation
- Once this determination is made, decide which tool is most efficient--mental arithmetic, paper and pencil, calculator, or computer

E. Students will use computation, estimation, and proportions to solve problems.

- They will be able to:
- Apply to new problem-solving contexts operations using scientific notation and integers
- Apply proportions to problem-solving situations

F. Students will use estimation to check the reasonableness of results.

They will be able to:

- Validate answers by checking actual solutions against estimates

V. PATTERNS AND FUNCTIONS

Students will organize data into lists or tables to find patterns contained in these data.

A. Students will describe, extend, analyze, and create a wide variety of patterns.

They will be able to:

- Apply arithmetic, power, and geometric sequences to real-life problems
- Explain how a specific problem illustrates a familiar pattern (i.e., Fibonacci sequences, Pascal's Triangle, or a sequence of squares)

B. Students will describe and represent relationships with tables, graphs, and rules.

They will be able to:

- Collect data, organize a table, and graph this data; later, graphing software or graphing calculators can be used to apply these procedures

C. Students will analyze functional relationships to explain how a change in one quantity results in a change in another.

They will be able to:

- Make a table using a function rule, and predict other results based on the pattern displayed

D. Students will use patterns and functions to represent and solve problems.

They will be able to:

- Use numbers, diagrams, block arrangements to determine a pattern and the rule derived from it

VI. ALGEBRA

Students will be able to translate English expressions into algebraic form.

A. Students will understand the concepts of variable, expression, and equation.

They will be able to:

- Translate English expressions into numerical expressions (the product of two tenths and fifty = $.2 \times 50$)
- Given a value of a variable, evaluate an algebraic expression (if $d = 5$, $d + d = 10$)

B. Students will represent situations and number patterns with tables, graphs, verbal rules, and equations and explore the interrelationships of these representations.

They will be able to:

- Find the solution of a problem with two variables by making a data table and/or a graph

C. Students will analyze tables and graphs to identify properties and relationships.

They will be able to:

- Read and interpret speed/time and distance/time graphs
- Compare rates of speed by comparing slopes of lines
- Relate a linear equation to its graph
- Determine whether data shows a direct or an inverse relationship

D. Students will develop confidence in solving linear equations using concrete, informal and formal methods.

They will be able to:

- Create scatter plots and graphs and interpret the functional relationship represented by these plots and graphs
- Write equations in the form $y = mx + b$
- Use tables, graphs, and equations to prepare both verbal and written descriptions of motion

E. Students will investigate inequalities and non-linear equations informally.

They will be able to:

- Graph information from concrete experiments: (i.e., graph of two different rates of pay to illustrate an inequality)
- Solve linear inequalities

F. Students will apply algebraic methods to solve a variety of real-world and mathematical problems.

They will be able to:

- Use formulas in an electronic spreadsheet to calculate price, cost, and income
- Work backwards to find starting point(s) when given problems with outcomes
- Apply algebraic expressions to solve problems of area, perimeter, and volume
- Use equations to develop the relationships among distance, time, and speed

VII. STATISTICS

Students will learn to find measures of central tendency

A. Students will systematically collect, organize, and describe data on paper and electronically.

They will be able to:

- Select data appropriate for a histogram
- Accurately determine medians, means, and ranges

B. Students will construct, read, and interpret tables, charts, and graphs.

They will be able to:

- Draw linear graphs
- Make and compare stem-and-leaf plots
- Use database programs to sort information in various ways
- Draw a histogram that uses convenient intervals
- Construct double bar graphs on graph paper or electronically

C. Students will make inferences and convincing arguments that are based on data analysis.

They will be able to:

- Draw conclusions that are based on data

- Make conjectures prior to data collection and modify these predictions as data are collected.

D. Students will evaluate arguments that are based on data analysis.

They will be able to:

- Describe what makes particular graphs misleading

E. Students will develop an appreciation for statistical methods as powerful means for decision-making.

They will be able to:

- Explain why one display is more appropriate than others in answering a question
- Demonstrate the strengths and limitations of different graphs
- Study and analyze examples of advertising that are based on data analysis
- Explain why an average should be reported
- Demonstrate what average is most appropriate in a given situation
- Translate a real-life problem into a statistical problem

VIII. PROBABILITY

Students will use probability techniques, including tree diagrams and the Permutation Theorem, to determine the number of possible permutations or orderings of events.

A. Students will model situations by devising and carrying out experiments or simulations to determine probabilities.

They will be able to:

- Use symbolic notation to express permutations
- Determine whether events are independent or dependent

B. Students will model situations by constructing a sample space to determine probabilities.

They will be able to:

- Draw a tree diagram to illustrate all possibilities in an experiment involving dependent events.
- Compare the convenience of using tree diagrams and tables to display the set of all possibilities.

C. Students will appreciate the power of using a probability model by comparing experimental results with mathematical expectations.

They will be able to:

- Demonstrate that the measure of certainty or uncertainty varies as more data are collected (i.e., batting average through a season)
- Determine theoretical probabilities
- Compare experimental results to mathematically derived probabilities.

D. Students will make predictions that are based on experimental or theoretical probabilities.

They will be able to:

- Apply proportional reasoning to predict outcomes with a large number of tries

IX. GEOMETRY

Students will explore the characteristics of two-dimensional shapes, their transformations, and their applications in coordinate geometry

A. Students will identify, describe, compare and classify geometric figures.

They will be able to:

- Explore and describe the properties of points and lines

B. Students will visualize and represent geometric figures with special attention to developing spatial sense.

They will be able to:

- Given a drawing of a sphere, prism, pyramid, cylinder, or cone, identify the attributes of the figure
- Given the attributes of a sphere, prism, pyramid, cylinder, or cone, make a construction with manipulatives, paper folding, etc.
- Given the attributes of a sphere, prism, pyramid, cylinder, or cone, make a drawing of the figure with paper/pencil or computer
- Given a simple figure made from Cuisenaire rods, double its dimensions and construct the new figure
- Find the surface area and volume of a figure by direct measurement (cover with sq. cm. paper for surface area; fill solid for volume) and compare with measurement by formula
- Investigate properties of circles and construct circles with a compass

C. Students will explore transformations of geometric figures.

They will be able to:

- Given half a figure, draw the whole figure by reflecting original half along a line
- Slide original figure and find new coordinates of this same figure
- Given coordinates of triangles, rectangles, trapezoids, expand, contract, and rotate these figures 180 degrees, using paper/pencil or computer (i.e., Geometer's Sketchpad, Geometry Inventor)

D. Students will represent and solve problems using geometric models.

They will be able to:

- Use models to solve arithmetic problems involving linear, square, and cubic measurements

E. Students will develop the ability to understand and apply geometric properties and relationships.

They will be able to:

- Given a set perimeter, maximize the rectangular area it can contain (using whole number increments)
- Demonstrate with various cylinders and string the relationship between circumference and diameter
- Understand pi and use it in order to determine the circumference and area of a circle

F. Students will develop an appreciation of geometry as a means of describing the physical world.

They will be able to:

- Show real-life examples of flips and turns, such as Tetris maneuvers

X. MEASUREMENT

Students will use the metric and traditional systems to measure length, perimeter, area, and volume. They will incorporate scientific notation to designate very large and very small dimensions.

A. Students will extend their understanding of the process of measurement.

They will be able to:

- Use the appropriate system of measurement in a real-life problem situation (i.e., length, volume, capacity, mass, scientific notation)
- Find angle measurements of supplementary, complementary, vertical, and corresponding and alternate interior angles

B. Students will estimate, make, and use measurements to describe and compare phenomena.

They will be able to:

- Use scientific notation to understand large distances and magnifications of the invisible world

C. Students will select appropriate units and tools to measure to the degree of accuracy required in a particular situation.

They will be able to:

- Construct a scale model of a house or of the solar system, using proportional reasoning; or use word processing to write a description of how a model of the solar system might be constructed.

D. Students will understand the structure and use of systems of measurement.

They will be able to:

- Write scientific notation, and apply it to problems about population, distance between planets, etc.

E. Students will extend their understanding of the concepts of perimeter, area, volume, angle measure, capacity, and weight and mass.

They will be able to:

- Demonstrate that all polygons can be partitioned into triangles
- Determine the sum of the interior angles of any polygon

F. Students will develop the concepts of rates and other derived and indirect measurements.

They will be able to:

- Use knowledge of similar triangles to measure heights of inaccessible objects

G. Students will develop formulas and procedures for determining measures to solve problems.

They will be able to:

- Demonstrate that the area of a figure does not change if it is partitioned and rearranged (i.e., trapezoid, hexagon)

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Mathematics Learning Expectations for Grade 8

I. PROBLEM SOLVING

Students will select from among a variety of strategies the appropriate means to solve a given problem.

A. Students will develop the ability to use problem-solving approaches to investigate and understand mathematical content.

They will be able to:

- Determine the most appropriate strategy for a given type of problem
- Write complete equations that represent problem situations with one or two variables
- Write in symbolic form a situation involving two variables
- Solve problems based on systems of equations

B. Students will develop the ability to formulate problems from situations within and outside mathematics.

They will be able to:

- Translate written information into algebraic notation

C. Students will develop and apply a variety of strategies to solve problems, with emphasis on multi-step and non-routine problems.

They will be able to:

- Select and use models or algorithms for solving complex problems
- Decide whether to use mental arithmetic, paper and pencil, or technology

D. Students will develop the ability to verify and interpret results with respect to the original problem situation.

They will be able to:

- Write symbolic representations that describe problem situations
- Explain how symbolic, visual, and graphical representations can communicate the same problem situation

E. Students will develop the ability to generalize solutions and strategies to new problem situations.

They will be able to:

- Generate questions and determine their relevance in solving a problem
- Justify in discussion a selection of strategy, and modify this choice based on new arguments presented by others

F. Students will develop the ability to acquire confidence in using mathematics meaningfully.

They will be able to:

- Demonstrate initiative in identifying real-world problems to investigate
- Make convincing arguments to justify hypotheses or results

II. NUMBER RELATIONSHIPS

Students will expand their knowledge of the number system to include scientific notation and rational numbers to solve more sophisticated problems.

A. Students will develop the ability to understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential and scientific notation) in real-world and mathematical problem situations.

They will be able to:

- Determine equivalent fractional form of numbers with negative exponents
- Determine when to use different equivalent forms
- Convert slope ratio (a fraction) to percent grade and tangent ratio (a decimal)

B. Students will develop number sense for whole numbers, fractions, decimals, integers, and rational numbers.

They will be able to:

- Determine when to use different forms of a whole number (+8, $\frac{8}{1}$, and 8.0 when subtracting integers, adding fractions, or labeling coordinate axes with rational numbers)

C. Students will develop the ability to understand and apply ratios, proportions and percents in a wide variety of situations.

They will be able to:

- Apply ratios to the concept of slope
- Calculate percent increase or decrease in different situations

D. Students will develop the ability to investigate relationships among fractions, decimals and percents.

They will be able to:

- Determine equivalent forms when given a fractional representation involving a variable ($\frac{1}{2}x = x/2 = .5x = .50x = 0.5x = 50\%x$)

E. Students will develop the ability to represent numerical relationships in one- and two-dimensional graphs.

They will be able to:

- Make a graph showing changes in slope as a result of increase or decrease, or change of direction
- Use paper and pencil, computer, or graphing calculator to represent changes in slope
- Use number lines to determine and display absolute values of integers
- Sketch a given equation representing either an arithmetic or geometric sequence

III. SYSTEMS AND NUMBER THEORY

Students will extend their understanding of rational numbers as they apply concepts of primes, factors, and multiples to simplify arithmetic and algebraic expressions.

A. Students will develop the ability to understand and appreciate the need for numbers beyond the whole numbers.

They will be able to:

- Solve for the variable in a second degree equation
- Apply the Pythagorean Theorem to determine lengths of sides of triangles.

B. Students will develop and use order relations for rational numbers (i.e., fractions, decimals, integers)

They will be able to:

- Describe what is between two negative integers (-2 and -3) or negative powers (3^{-2} and 3^{-3})
- Summarize the laws of exponents
- Construct a typical graph of an exponential function

C. Students will develop the ability to extend their understanding of whole number operations to fractions, decimals, integers, and rational numbers

They will be able to:

- Apply distributive property to solve equations and to write expressions
- Solve equations that involve more than one form of number representation $\{5+(7/8)\}$

D. Students will develop the ability to understand how the basic arithmetic operations are related to one another.

They will be able to:

- Use multiple steps to solve for the variable when given an equation with a single variable, ($3x + 7 = x - 5$), thereby, using the inverse of the given operations

E. Students will develop and apply number theory concepts, e.g., primes, factors, and multiples) in real-world and mathematical problem situations.

They will be able to:

- Recognize patterns in Fibonacci sequences, Pascal's triangle, and in figurate numbers
- Identify patterns as they appear in real-world situations (Fibonacci sequences, Pascal's triangle, the Golden Rectangle, figurate numbers)

IV. COMPUTATION AND ESTIMATION

Students will determine if an estimate or exact answer is required, and then select from among paper and pencil, mental arithmetic, or technology to solve the given problem.

A. Students will develop the ability to compute with rational numbers (i.e. fractions, decimals, integers).

They will be able to:

- Factor polynomials with integral coefficients
- Factor polynomials with decimal coefficients
- Factor polynomials with fractional coefficients
- Use integer computation within equations to combine like terms

B. Students will develop, analyze, and explain procedures for computation and techniques for estimation.

They will be able to:

- Practice rules for order of operations with symbolic notation
- Estimate the volume and surface area of given real-life shapes

C. Students will develop, analyze, and explain methods for solving proportions.

They will be able to:

- Analyze a real-life situation involving proportionality and describe it in numerical terms

D. Students will select and use an appropriate method for computing from among mental arithmetic, paper-and-pencil, calculator, and computer methods.

They will be able to:

- Decide if an exact answer is required, or if an estimate is sufficient, when given a problem situation
- Once this determination is made, students must decide which tool is most efficient -- mental arithmetic, paper and pencil, calculator, or computer

E. Students will use computation, estimation, and proportions to solve problems.

They will be able to:

- Approximate square roots of numbers that are not perfect squares (6 Å10 feet is approximately what distance?)
- Apply proportions to non-routine problem-solving situations
- Recognize the application of Pythagorean theorem

F. Students will use estimation to check the reasonableness of results.

They will be able to:

- Validate answers by checking actual solutions against estimates

V. PATTERNS AND FUNCTIONS

Students will organize data into lists or tables to find patterns contained in these data.

A. Students will describe, extend, analyze, and create a wide variety of patterns.

They will be able to:

- Classify and/or characterize a given sequence as arithmetic, geometric, exponential, or Fibonacci
- Generalize patterns from observed data

B. Students will describe and represent relationships with tables, graphs, and rules.

They will be able to:

- Describe rules algebraically
- Translate a problem given in words or in a diagram into a table, graph, and expression or equation

C. Students will analyze functional relationships to explain how a change in one quantity results in a change in another.

They will be able to:

- Examine distance formula to determine inverse relationships created by data (Examine data generated using $D = RT$, and predict results)
- Use the tangent ratio and the inverse tangent to find unknown angle measures and side lengths in right triangles

D. Students will use patterns and functions to represent and solve problems.

They will be able to:

- Given a situation involving equations and inequalities, graph and interpret the data and write a summary based on the solution(s)
- Determine if a set of data shows linear or exponential growth and select a line of best fit to reflect these data.

VI. ALGEBRA

Students will be able to translate English expressions into algebraic form.

A. Students will understand the concepts of variable, expression, and equation.

They will be able to:

- Choose a familiar formula from science, solve for any of the variables.

B. Students will represent situations and number patterns with tables, graphs, verbal rules, and equations and explore the interrelationships of these representations.

They will be able to:

- Make a chart and translate data into an algebraic equation when given a problem with two variables

C. Students will analyze tables and graphs to identify properties and relationships.

They will be able to:

They will be able to:

- Given equations, tables and graphs, determine which equation or table represents which graph
- Compare graphs and tables to identify relationships
- Given tables and/or graphs of data, decide whether the relationships are direct or inverse
- Justify in writing the results of the investigations

D. Students will develop confidence in solving linear equations using concrete, informal and formal methods.

They will be able to:

- Recognize when a linear equation describes a situation, and write an appropriate equation to solve it
- Describe the major characteristics of a linear function

E. Students will investigate inequalities and non-linear equations informally.

They will be able to:

- Shade graphs to illustrate inequalities
- Make a graph of data representing geometric growth, and extrapolate possible values or outcomes

F. Students will apply algebraic methods to solve a variety of real-world and mathematical problems.

They will be able to:

- Given a situation, describe it algebraically, plot data, solve the problem and explain the process and results to peers

VII. STATISTICS

Students will learn to find measures of central tendency

A. Students will systematically collect, organize, and describe data on paper and electronically.

They will be able to:

- Select data appropriate for a histogram
- Accurately determine medians, means, and ranges

B. Students will construct, read, and interpret tables, charts, and graphs.

They will be able to:

- Draw linear graphs

C. Students will make inferences and convincing arguments that are based on data analysis.

They will be able to:

- Make conjectures prior to data collection and modify these predictions as data are collected.
- Draw conclusions that are based on data.

D. Students will evaluate arguments that are based on data analysis.

They will be able to:

- Anticipate problems confronted by researchers as they use various methods
- Demonstrate when a line graph has advantages over a bar graph
- Develop graphs using different scales to alter the message communicated
- Make data-based comparisons

E. Students will develop an appreciation for statistical methods as powerful means for decision-making.

They will be able to:

- Explain why an average should be reported

- Demonstrate what average is most appropriate in a given situation
- Study real-life examples (such as sports) that are based on data analysis
- Translate a real-life problem into a statistical problem

VIII. PROBABILITY

Students will use probability techniques, including tree diagrams and the Permutation Theorem, to determine the number of possible permutations or orderings of events.

A. Students will model situations by devising and carrying out experiments or simulations to determine probabilities.

They will be able to:

- Determine whether events are independent or dependent
- Conduct an experiment involving dependent events (picking cards from a deck, selecting different color socks)

B. Students will model situations by constructing a sample space to determine probabilities

C. Students will appreciate the power of using a probability model by comparing experimental results with mathematical expectations.

They will be able to:

- Demonstrate that the measure of certainty or uncertainty varies as more data are collected (batting average through a season)
- Determine theoretical probabilities

D. Students will make predictions that are based on experimental or theoretical probabilities.

They will be able to:

- Compare experimental results to mathematically derived probabilities
- Use a computer simulation to show how a greater number of trials can refine a probability model.

E. Students will develop an appreciation for the pervasive use of probability in the real world.

They will be able to:

- Study real-life examples such as sports, forecasting, and public policy that are based on data analysis (population growth, number of family farms, cost of consumer goods, etc.)

IX. GEOMETRY

Students will explore the characteristics of two-dimensional shapes, their transformations, and their applications in coordinate geometry

A. Students will identify, describe, compare and classify geometric figures.

They will be able to:

- Describe in terms of its attributes a given 3-dimensional shape of a sphere, prism, pyramid, cylinder, or cone.
- Explore and describe the properties of points and lines in a coordinate plane

B. Students will visualize and represent geometric figures with special attention to developing spatial sense.

C. Students will explore transformations of geometric figures.

They will be able to:

- Find line of symmetry and rotations of 3-dimensional figures such as cube, prism, cylinder, pyramid
- Find the axis of symmetry of a parabola that is the graph of a quadratic equation
- Find the distance between the axis of symmetry of a parabola and either of its x-intercepts
- Given triangles, rectangles, trapezoids, demonstrate rotations from 1° to 360° (Draw with paper and pencil or with a computer)
- Given original coordinates of triangles, rectangles, and trapezoids, identify and draw new coordinates when figure is reflected across the X or Y axis

D. Students will represent and solve problems using geometric models.

They will be able to:

- Use area model to solve simple algebraic equations (leg of a right triangle, given the area and the length of the other leg)
- Derive formulas for surface area and volume of 3-D shapes

E. Students will develop the ability to understand and apply geometric properties and relationships.

They will be able to:

- Demonstrate how scaling up or down in size affects length, surface area, and volume of an object

F. Students will develop an appreciation of geometry as a means of describing the physical world.

They will be able to:

- Describe how changes in volume of a polyhedron relate to the changes in the surface area
- Use graphical representations (table, graph) to communicate changes resulting from scaling a figure up or down in size
- Students will derive formulas such as the Pythagorean Theorem to determine dimensions, area, and volume of two- and three-dimensional figures.

X. MEASUREMENT

Students will use the metric and traditional systems to measure length, perimeter, area, and volume. They will incorporate scientific notation to designate very large and very small dimensions.

A. Students will extend their understanding of the process of measurement.

They will be able to:

- Use the appropriate system of measurement in a real-life problem situation (algebraically using formulas, Pythagorean theorem, mixtures, uniform motion, rates of change)

B. Students will estimate, make, and use measurements to describe and compare phenomena.

They will be able to:

- Use formulas to help predict outcomes of posed problems (rates of change, mixtures, uniform motion, increase in volume)

- Use the Pythagorean Theorem to measure legs or hypotenuse of a right angle
- Determine the unknown lengths and angles of a set of triangles when given angle, height and grade specifications of a hill

C. Students will select appropriate units and tools to measure to the degree of accuracy required in a particular situation.

They will be able to:

- Use the quadratic formula as a tool to measure distance or height (Determine the height of a thrown ball at any given point)
- Calculate and then compare with a graph the areas of rectangles that all have the same perimeter but have varying dimensions

D. Students will understand the structure and use of systems of measurement.

They will be able to:

- Compare temperatures using Fahrenheit, Celsius, and Kelvin scales
- Compare liquid measures using English and metric systems

E. Students will extend their understanding of the concepts of perimeter, area, volume, angle measure, capacity, and weight and mass.

They will be able to:

- Approximate the area of an irregular figure by covering the figure with a square grid and counting the whole squares within the figure as an inner measure and counting all squares that touch the figure anywhere as an outer measure (the actual measure is between these two)
- Determine the size of an angle, using a protractor or a Slope-o-meter interchangeably
- Use the tangent relationship to compare the lengths of two different triangles.

F. Students will develop the concepts of rates and other derived and indirect measurements.

They will be able to:

- Develop and apply formulas involving linear and exponential growth (compare simple and compound interest)

G. Students will develop formulas and procedures for determining measures to solve problems.

They will be able to:

- Generalize from a chart the relationship between Celsius and Fahrenheit temperatures, and develop the formulas to convert between Fahrenheit and Celsius temperatures
- Apply the formulas that relate Celsius and Fahrenheit temperatures
- Find a formula for rates of change

This is the end of the Grades 6-8 Mathematics Curriculum Overview, Cambridge Public Schools